Dense quark matter in nature

Mark Alford

Washington University
Saint Louis, USA

Outline

I Dense quark matter

color superconductivity

II Real-world quark matter

strange quark; weak equilibrium; neutrality

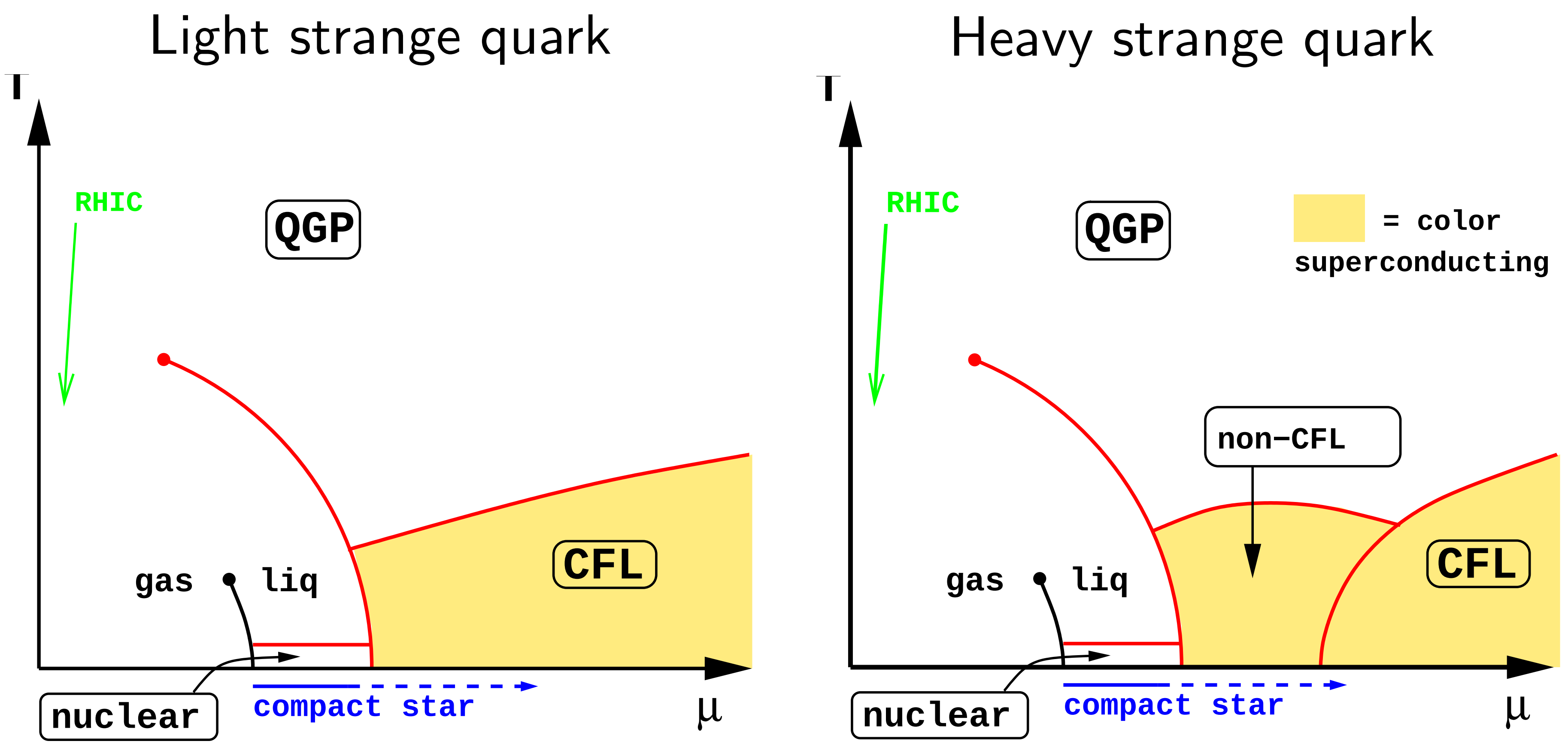
III Compact stars

Mass-radius relationship

IV Looking to the future

I. Dense quark matter

Conjectured QCD phase diagram

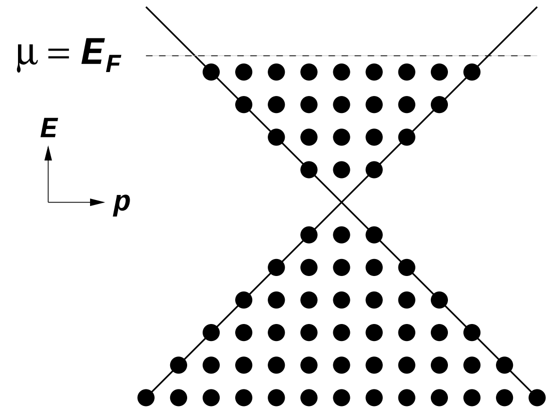


heavy ion collisions: chiral critical point and first-order line

compact stars: color superconducting quark matter core

Color superconductivity

At sufficiently high density and low temperature, there is a Fermi sea of almost free quarks. Attractive QCD interactions $\Rightarrow$ BCS *pairing* at the Fermi surface.



Simplest pairing patterns:

Three flavors ($m_s = 0$), color-flavor locking (CFL):

$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta$$

chiral symmetry broken

2+1 flavors (low m_s), modified CFL:

u-s and d-s pair differently from u-d chiral symmetry broken

2 flavors (high m_s), 2SC:

Strange quarks decouple, u and d pair,

$$\langle q_i^\alpha q_j^\beta \rangle \sim \varepsilon_{ij} \varepsilon^{\alpha\beta 3}$$

chiral symmetry unbroken

II. Real-world quark matter

In the real world, things are more complicated:

1. The strange quark mass is not infinite nor zero, but intermediate. It depends on density, and ranges between about 500 MeV in the vacuum and about 120 MeV at high density.
2. In a compact star there is time for weak equilibration to occur: neutrinos escape and flavor is not conserved.
3. Dense matter, eg in the core of compact stars, must be neutral with respect to all gauge charges: color and electromagnetism.

Color superconductivity with realistic strange quark mass

With $m_s \sim 120-500$ MeV, the pure CFL pairing pattern is distorted and perhaps broken. Approaches to studying this are:

1. Use a more general pairing ansatz, eg allowing d-u, d-s etc to pair with different strength.
2. Use an effective theory of the low-energy degrees of freedom, including flavor rotations of the CFL condensate ("pions" and "kaons").
3. Allow condensates with momentum: crystalline color superconductivity.

This is still a very active area of research.

Neutrality and weak equilibrium

- Weak equilibrium

$$
\begin{array}{lll}
u & \longrightarrow & d\ e^+\ \bar\nu \\
u & \longrightarrow & s\ e^+\ \bar\nu
\end{array}
\qquad\qquad
\begin{array}{rcl}
\mu_u & = & \mu - \frac{2}{3}\mu_e \\
\mu_d = \mu_s & = & \mu + \frac{1}{3}\mu_e
\end{array}
$$

- Electromagnetic neutrality

$$Q = \frac{\partial\Omega}{\partial\mu_e} = 0$$

[NB: But there may be a globally neutral mixture of positive nuclear matter with negative quark matter.]

- Color neutrality. In unitary gauge, number of red, green, blue quarks must be the same. The cost of projecting to a color singlet is then negligible. $T_3 = \mathrm{diag}(1, -1, 0)$, $T_8 = \mathrm{diag}(1, 1, -2)$

$$\langle Q_3\rangle = \frac{\partial\Omega}{\partial\mu_3} = 0 \qquad \langle Q_8\rangle = \frac{\partial\Omega}{\partial\mu_8} = 0$$

So a general calculation of the EoS for color-superconducting QM takes the form

1. Write down free energy $\Omega(\mu, m_s, \mu_3, \mu_8, \mu_e, \Delta)$
2. Solve

$$\text{Neutrality conditions:} \quad \frac{\partial \Omega}{\partial \mu_e} = \frac{\partial \Omega}{\partial \mu_3} = \frac{\partial \Omega}{\partial \mu_8} = 0$$

$$\text{Gap equations:} \quad \frac{\partial \Omega}{\partial \Delta} = 0$$

For a complicated pairing ansatz there may be many gap parameters Δ_i each with its own gap equation.

But for basic insight into the nature of real-world quark matter, we can treat the gap Δ as a parameter, and expand in powers of Δ/μ and m_s/μ. In this case the CFL pattern is generally favored over 2SC.

III. Compact stars

Signatures of color superconductivity in compact stars

Gaps in spectra affect Transport properties.

Pairing energy affects Equation of state.

Transport properties, mean free paths, conductivities, viscosities, etc.

1. Cooling by neutrino emission, neutrino pulse at birth
   (Page, Prakash, Lattimer, Steiner, `hep-ph/0005094`; Carter and Reddy, `hep-ph/0005228`)
2. Gravitational waves: r-mode instability
   (Madsen, `astro-ph/9912418`)
3. Glitches and crystalline ("LOFF") pairing
   (Alford, Bowers, Rajagopal, `hep-ph/0008208`)

Equation of state (mass-radius relationship)

Pressure of quark matter relative to hadronic vacuum

$$p = (\cdot)\, \mu^4 - (\cdot)\, {m_s}^2 \mu^2 + (\cdot)\, \Delta^2 \mu^2 - B$$

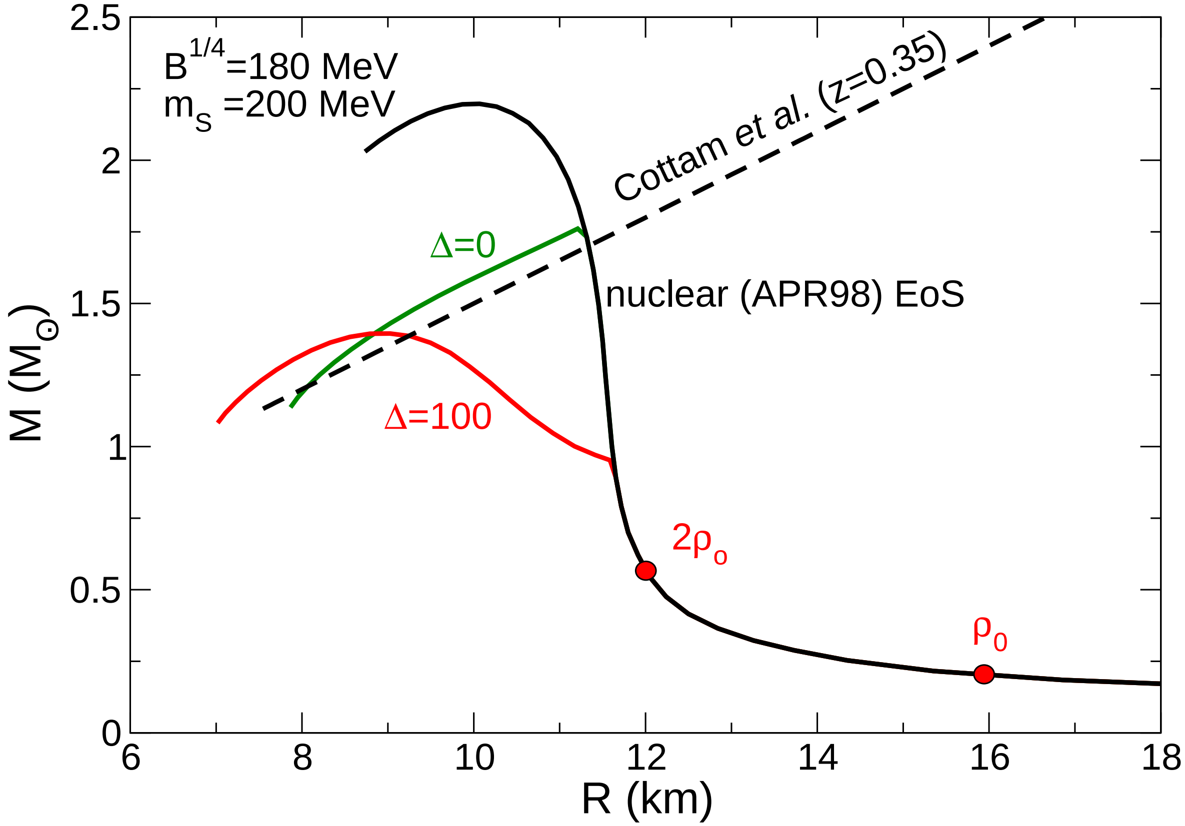
If bag constant is such as to bring quark matter close to stability, a superconducting gap Δ may have large effects on the mass-radius relationship.

Alford and Reddy, nucl-th/0211046; bag model: hybrid stars and mixed phases

Lugones and Horvath, astro-ph/0211638; bag model: strange stars

Baldo et al, nucl-th/0212096; NJL model: no stable hybrid stars

Mass-radius relationship: effect of color SC at fixed B



Green and red lines correspond to *hybrid* stars: quark matter core, nuclear matter mantle.

Alford and Reddy, nucl-th/0211046

Color superconductivity ($\Delta > 0$) makes hybrid stars more stable at fixed bag constant B.

Hybrid stars seem smaller and lighter than pure nuclear matter stars.

Including perturbative corrections in QM EoS

pressure $\quad p = (1-c)\,(\cdot)\,\mu^4 - (\cdot)\,{m_s}^2\mu^2 + (\cdot)\,\Delta^2\mu^2 - B$

Parameters:

c: Correction due to QCD interactions between quarks.
Estimated $c \approx 0.4$ (Fraga, Pisarski, Schaffner-Bielich, hep-ph/0101143)

m_s: Strange quark mass. Actually depends on μ. $m_s = 150$–400 MeV.

Δ: Color-superconducting gap. Depends on μ. $\Delta \sim 10$–100 MeV
We assume the CFL pairing pattern.

B: Bag constant. May depend on μ. $B^{1/4} \sim 150$–200 MeV

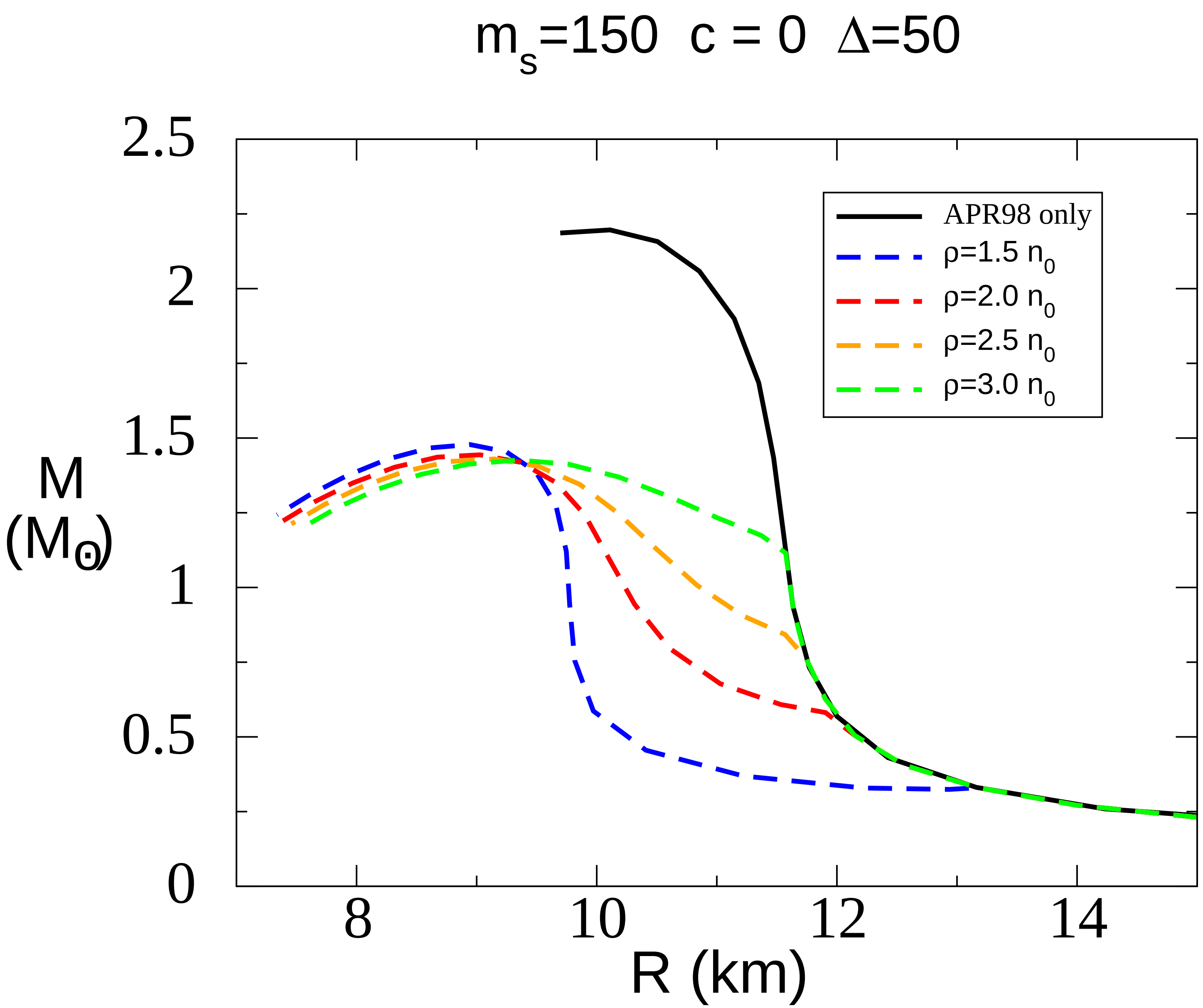
Survey of the effect of the parameters of QM EoS on $M(R)$

If the bag constant B were known, all the other parameters, including Δ would have a strong effect of $M(R)$, in part because they effectively renormalize B.

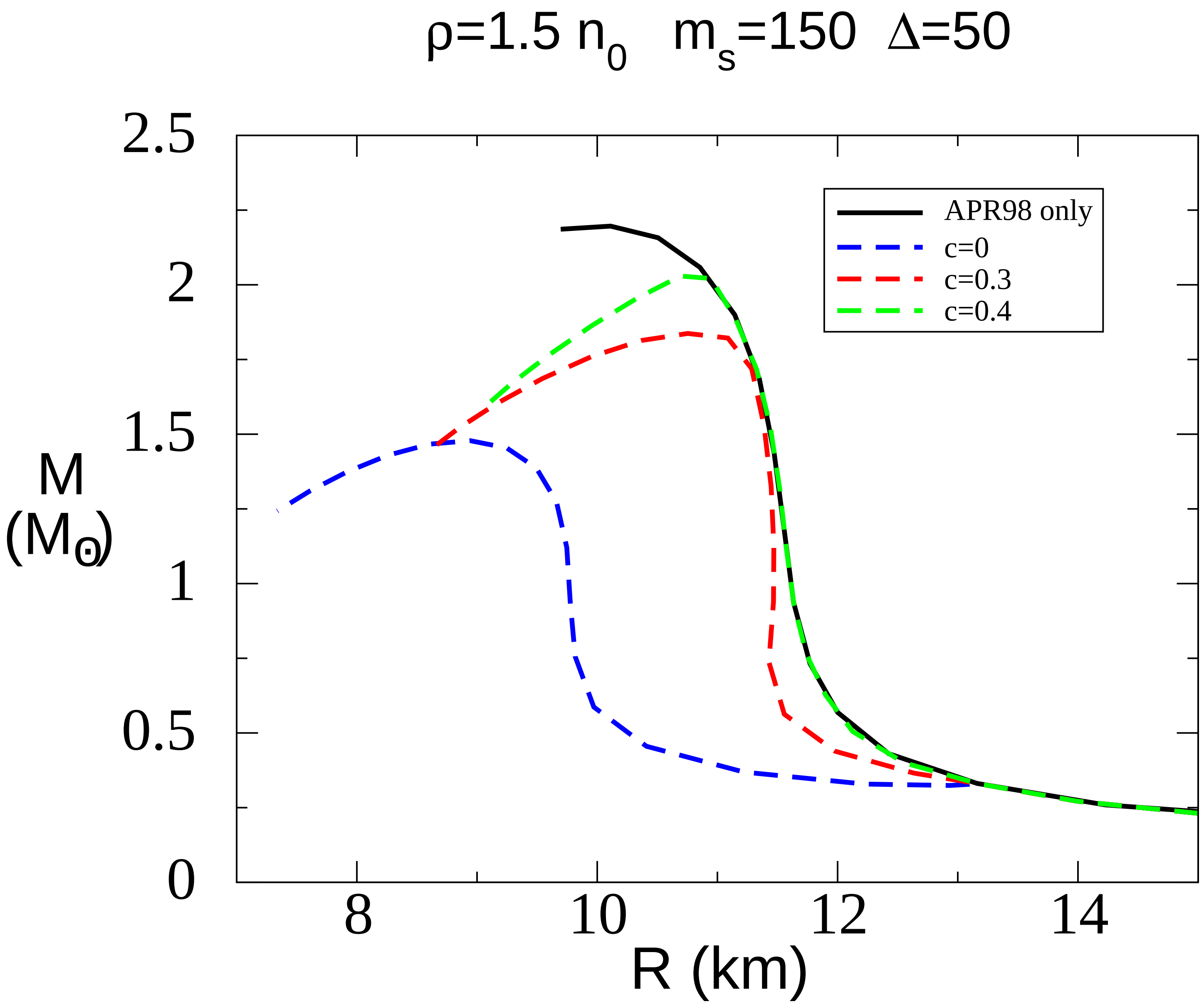
Alternatively, let us treat B as completely unknown, and for each set of parameters choose it to fix a physical quantity, the density ρ at which the NM$\longrightarrow$QM transition occurs.

So the parameters of QM are ρ, c, m_s, Δ.

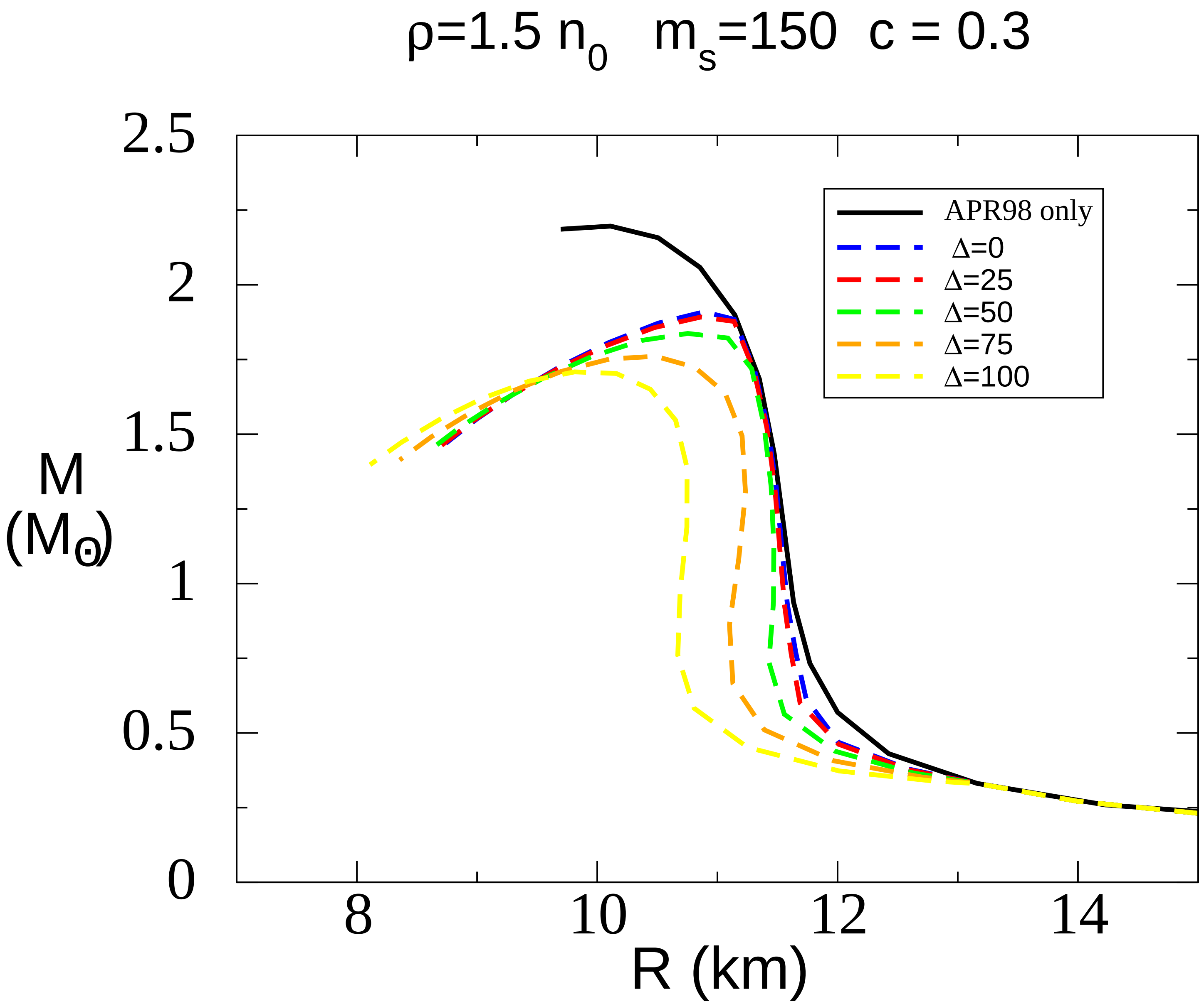
$M(R)$, varying transition density ρ



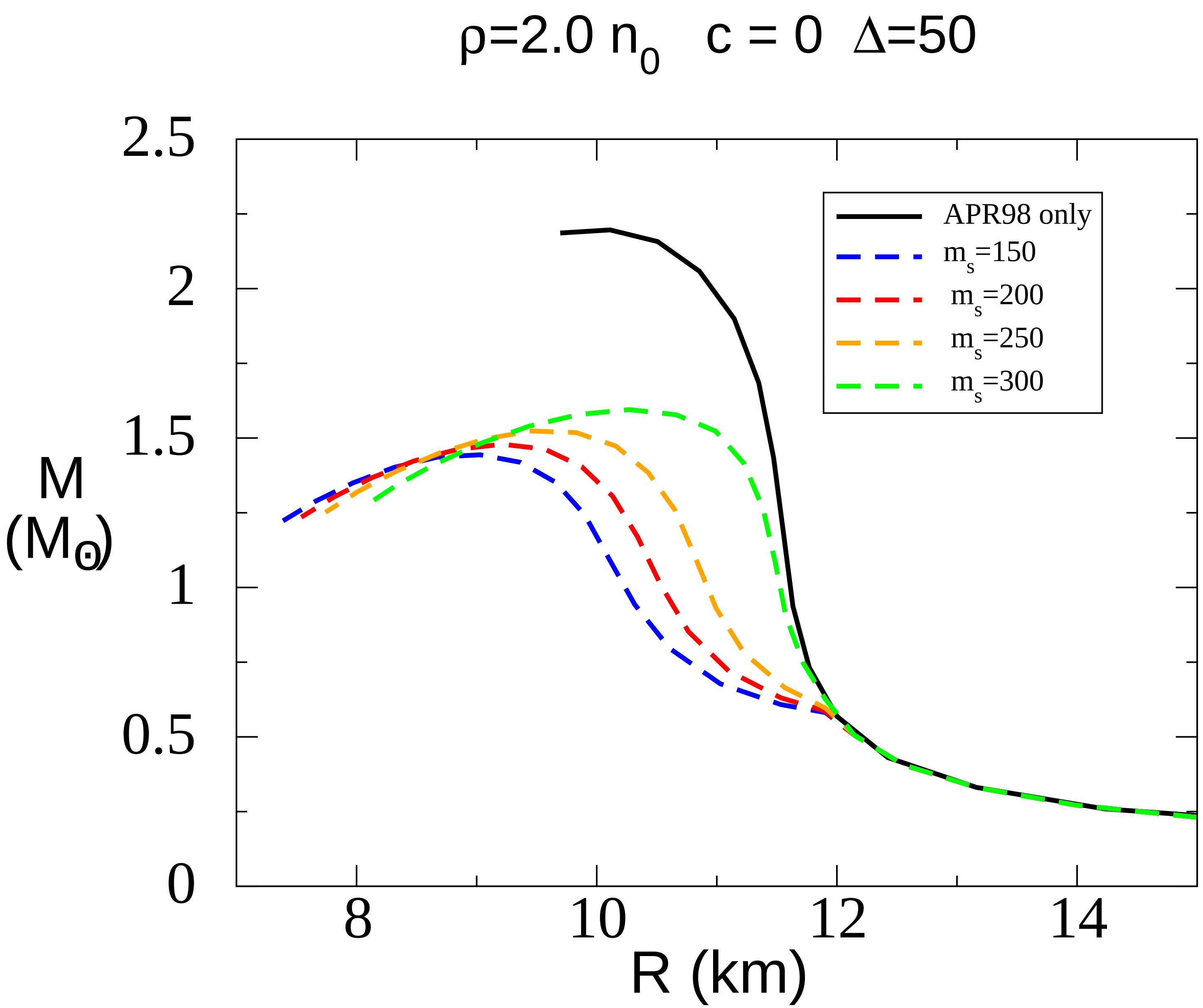
$M(R)$, varying "perturbative" correction c



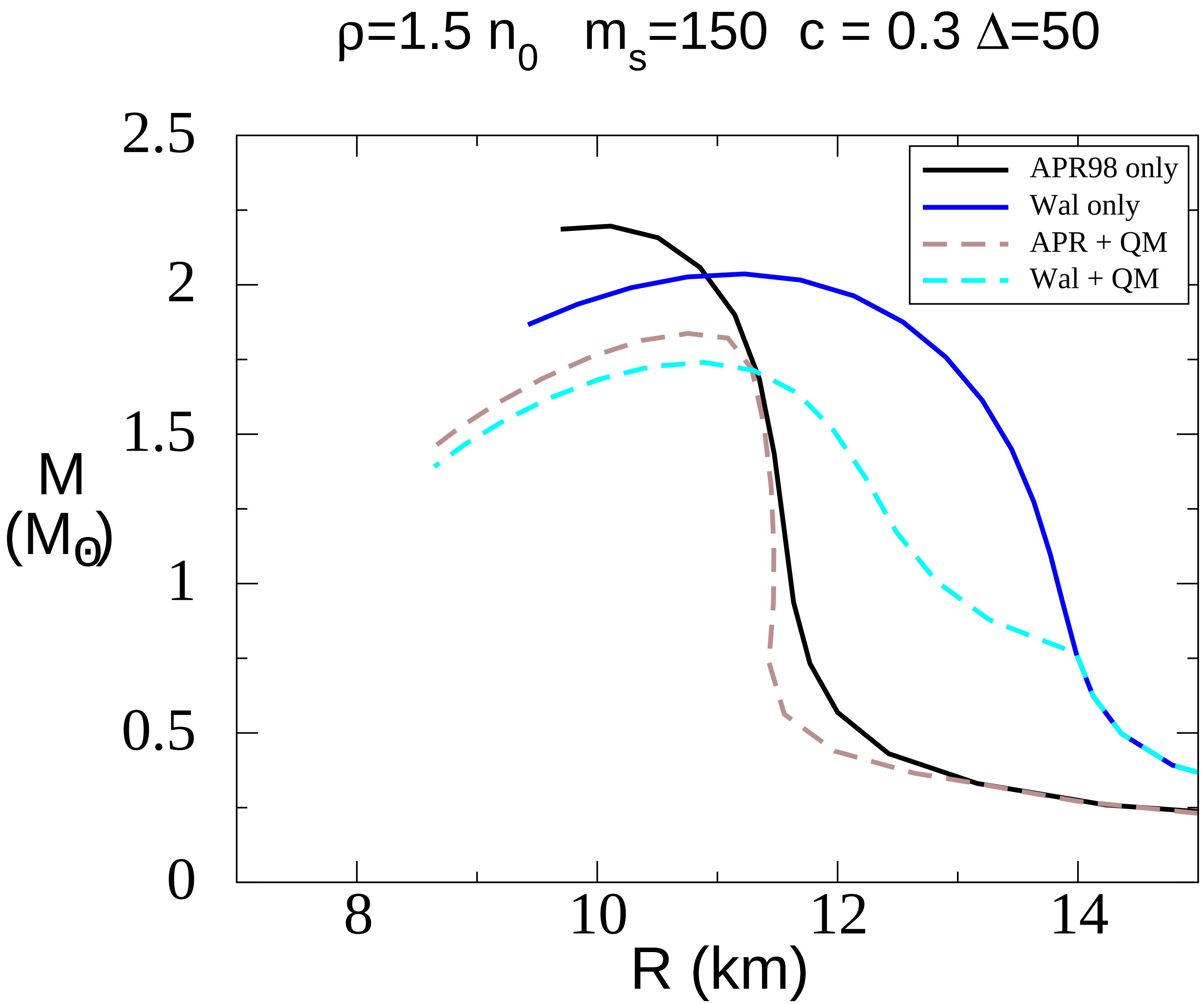
$M(R)$, varying color superconducting gap Δ



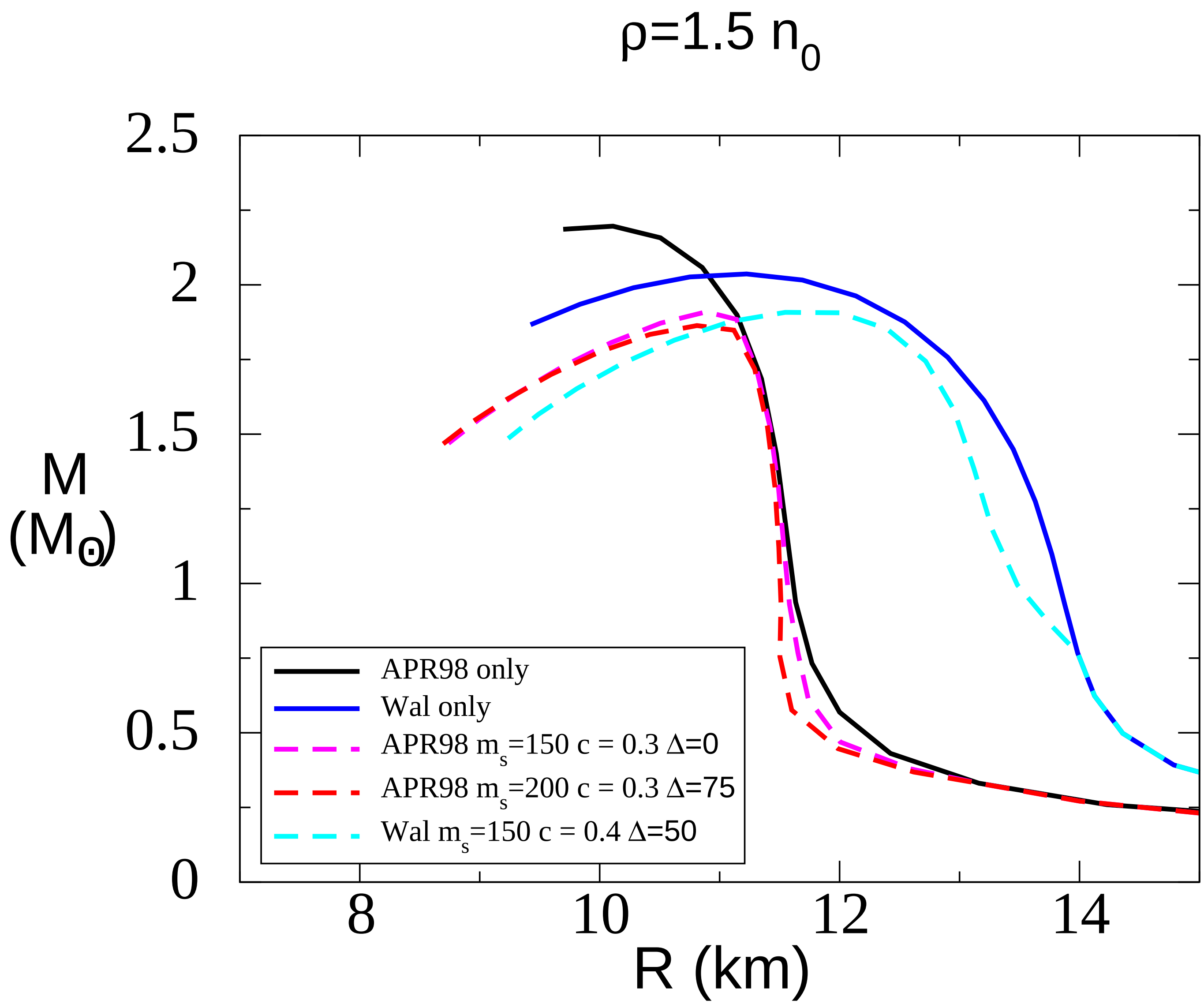
$M(R)$, varying strange quark mass m_s



$M(R)$, for different nuclear matter EoS



How heavy can hybrid stars be?



For $c \approx 0.3$, hybrid stars can reach $M \approx 1.9\ M_\odot$.

$M(R)$ measurements and quark matter

What would rule out quark matter?

Observed mass $M \gtrsim 2\ M_\odot$

(Based on bag model with pert corrections and color SC, ignoring mixed phases).

What would indicate the presence of quark matter?

Difficult. Regions of M-R space that cannot be reached by any nuclear matter EoS also cannot be reached by hybrid NM-QM EoS.

What would indicate the presence of color superconducting quark matter?

More difficult. Even if we found an $M(R)$ characteristic of quark matter, we would need an independent determination of the bag constant to claim that it was color-superconducting.

IV. Looking to the future

- Neutron-star phenomenology of color superconducting quark matter:
  - Structure: nuclear-quark interface
  - Crystalline phase and glitches
  - Vortices but no flux tubes
  - Effects of gaps in quark spectrum
    - conductivity and emissivity (neutrino cooling)
    - shear and bulk viscosity (r-mode spin-down)
- Particle theoretic questions:
  - Response of CFL to m_s: gapless CFL, kaon condensation
  - Better weak-coupling calculations, include vertex corrections
  - Go beyond mean-field, include fluctuations.